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Classical and Quantum Limitations on Energy Consumption in Computation

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Fundamental limitations on the energy dissipated during one elementary logical operation are discussed. A model of a real physical device (parametric quantron) based on the Josephson effect in superconductors is used throughout the discussion. This device is shown to be physically reversible, and moreover it can serve as the elementary cell of a logically reversible computer, both these properties being necessary to achieve the fundamental limits of energy dissipation. These limits due to classical and quantum statistics are shown to lie well below the earlier estimates, $k_B T$ and $\hbar/\tau$, respectively.

1. INTRODUCTION

Minimum energy dissipation is one of the basic problems of physics of computation. During the last two decades, this problem has been a subject of extensive discussion; quite adequate reviews of the discussion have been given by Keyes (1975) and Landauer (1976). The understanding achieved during the discussion is not quite complete for nowadays, when new nonlinear elements—superconducting Josephson junctions [see, for example, monographs by Kulik and Yanson (1972), Solimar (1972), and Likharev and Ulrich (1978)]—have become available for a computer design.

In fact, these junctions are potentially ideal computer elements in many aspects, including energy consumption. A Josephson junction is essentially a nonlinear energy-storage (reactive) element. Nonlinearity of such an element can be characterized by nonquadratic dependence of its potential energy U on some physical variable x, describing the state of the element. If $U(x)$ has at least two minima, the element can be used for storage of information without permanent energy dissipation. Moreover, if the shape of the "potential well" $U(x)$ is flexible enough, the element can be used for the information processing (Landauer, 1961). It is believed that such nonlinear

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energy-storage elements (rather than permanently energy-consuming elements like transistors) can enable one to achieve the fundamental limits of energy dissipation.

In this paper, an analysis of the energy consumption by nonlinear energy-storage elements is given. In Section 2, we will demonstrate that a sufficiently flexible potential well can be realized by the parametric quantron (PQ), a very simple device using Josephson junctions. Changing the shape of $U(x)$ by external "forces" in a proper way, this device can be switched from one stable state to another in a reversible way, i.e., without any rapid "jumps" of the variable x. This physical reversibility provides gradual decrease of energy dissipation when the process period τ is increased.

In Section 3, we will show that a logically reversible computation proposed by Bennett (1973) can be performed using PQs interconnected and biased properly, thus excluding the "entropy loss" $W_c = k_B T \ln 2$ of energy (per an elementary logic operation). The results of Sections 2 and 3 show PQ to be an ideal computer element in its aspect of low energy dissipation W at given parameters like temperature T, eigenfrequency ω, relaxation time $\hat{\tau}$, operation period τ, and permissible error probability p. This allows one to analyze the fundamental limitations of W using the PQ model of a logical cell.

Classical limitations due to finite thermal fluctuation energy $k_B T$ are analysed in Section 4. The lower bound of W is found to be proportional to T, τ^{-1}, and Q^{-1}, where Q is the element's quality factor.

For superconductor devices like PQ, with their high eigenfrequencies ω, and realizability at low temperatures, one can really come to the situation when $k_B T$ becomes lower than $\hbar\omega$. In Section 5, we will demonstrate that the quantum-mechanical effects, particularly the macroscopic quantum tunneling, provide the bound for energy consumption in this situation. The bound, however, is found to be much lower than the earlier estimate $W \gtrsim \hbar/\tau$ by Bledsoe (1961), Marko (1965), and Likharev (1977).

In the conclusion, the results for the minimum energy dissipation in energy-storage computer elements are summarized and discussed.

2. PARAMETRIC QUANTRON AS PHYSICALLY REVERSIBLE DEVICE

Consider a superconducting ring of inductance L enclosed by a Josephson junction (Figure 1a). This device ("single-junction superconducting interferometer," or "ac SQUID," or "simple quantron") is well known owing to its wide applications in superconducting quantum magnetometers. To describe the features of the device, we will write down its potential

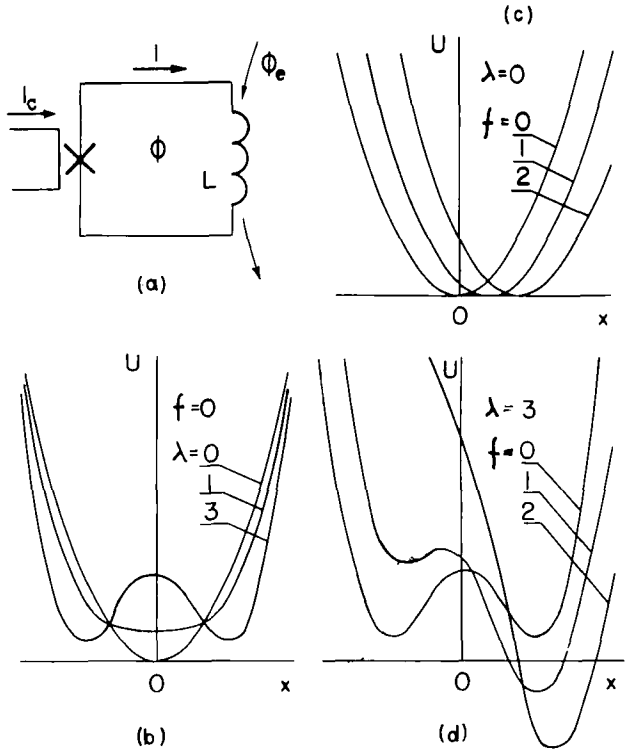

Fig. 1. Parametric quantron (a) and its potential energy U as a function of coordinate x (normalized magnetic flux Φ) at various values of parameters $\lambda=(2\pi/\Phi_0)I_M L$ and $f=(2\pi/\Phi_0)\Phi_e-\pi$ (b)–(d). Cross denotes a Josephson junction with the critical current I_M controlled by current I_c.

energy U, which is a sum of the magnetic energy of the ring

$$U_M = LI^2/2 \tag{1}$$

and the coupling energy of the Josephson junction

$$U_J = -(\Phi_0/2\pi)I_M \cos\varphi \tag{2}$$

Here I_M is the junction critical current, $\Phi_0=\hbar/2e\approx 2\times 10^{-15}$ Wb is the magnetic flux quantum, and φ is the superconducting order-parameter phase difference, related directly to the net magnetic flux Φ in the ring,

$$\Phi = (\Phi_0/2\pi)\varphi \tag{3}$$

Taking into account the contribution of the external magnetic field flux Φ_e to the net flux Φ,

$$\Phi = \Phi_e - LI \tag{4}$$

we obtain the following expression for the system's potential energy $U = U_M + U_J$:

$$U = (\Phi_0/2\pi)^2 L^{-1}\left[(x-f)^2/2 + \lambda \cos x\right]$$

$$x = \varphi - \pi, \qquad f = \varphi_e - \pi, \qquad \varphi_e = (2\pi/\Phi_0)\Phi_e \tag{5}$$

where λ is a basic dimensionless parameter of the device:

$$\lambda = (2\pi/\Phi_0) L I_M \tag{6}$$

Figures 1b–1d show the $U(x)$ dependence (5) for a number of values of λ and f. One can see that if the critical current I_M is large enough ($\lambda > 1$), x can have several stable states. Thus, the quantron can be used for the energy storage without any permanent power dissipation (Likharev, 1975). The information is encoded by the value of variable x, i.e., by the value of magnetic flux Φ trapped inside the interferometer ring. At, say, $f = 0$ and $\lambda = 3$ (Figure 1b) the equilibrium (static) value $x = +x_0 \approx 2.3$ can denote binary unity, while state $x = -x_0$ denotes binary zero.

Figures 1b–1c show that the effect of parameters λ and f on the shape of the potential well is rather different. The parameter λ changes the height of the energy barrier at $x \approx 0$ (for the sake of simplicity, we will limit ourselves to the case of not very large values of λ, in order that energy barriers at $x \approx 2\pi n$ do not appear)—Figure 1b. On the other hand, the parameter f governs the value of the well asymmetry. If $\lambda \ll 1$, f merely shifts the single minimum along the x axis—Figure 1b. If $\lambda \gg 1$, f retains both minima of the energy at almost the same position on the x axis, but produces a difference $\Delta U \approx 2fx_0$ between them, so that the upper local minimum finally disappears—Figure 1d.

It is clear now that with the control of not only f (external flux), but also λ, we would obtain a very flexible potential well. This is the basic idea of the parametric quantron, PQ (Likharev, 1977), where a simple Josephson junction is replaced by a junction with critical current controlled by current I_c. Such a controllable Josephson junction can be realized using either a distributed structure or a set of parallel-connected lumped junctions (see, for example, Solimar, 1972). In the simplest case of two similar lumped

junctions, connected in parallel by low-inductance superconducting leads,

$$I_M = 2I_0 \cos\left[(2\pi/\Phi_0)MI_c\right] \tag{7}$$

where I_0 is the critical current of a single junction, and M is the mutual inductance of the control current line carrying I_C and the loop formed by two junctions.

Flexibility of the energy well of the PQ makes it possible to switch it from one stable state into another without any rapid irreversible "jumps" of the variable x. To accomplish this, we start from the symmetric well realized in the absence of bias: $f = 0$ (Figure 2a), and apply a small bias corresponding in sign to the state of the system (Figure 2b). Now we can suppress the

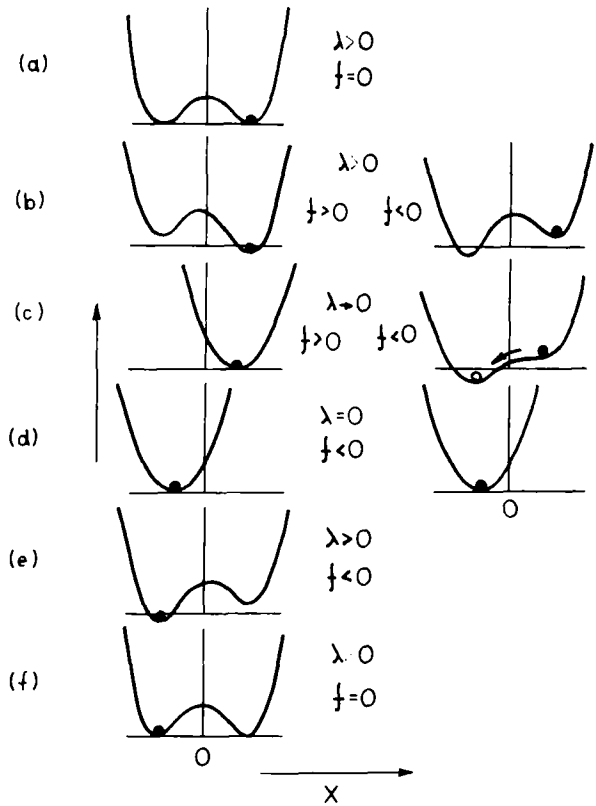

Fig. 2. Deformation of $U(x)$ dependence during the process of reversible switching PQ from one stable state to another one—left column. Right column demonstrates the rapid jump of coordinate x in the case of improper polarity of the preliminary biasing f.

energy barrier by decreasing λ (Figure 2c), change smoothly f to the same value of the opposite sign (Figure 2d), and restore the energy barrier by increasing λ (Figure 2e). Removing the bias, we come back to the initial shape of the potential well, but with the opposite state of the device (Figure 2f).

Note that if the bias f before the energy barrier suppression is chosen in a wrong way, i.e., of polarity opposite to that of the PQ state, the removal of barrier results in a rapid jump of the variable x when the upper local minimum of $U(x)$ disappears (Figure 2, right column). This jump is accompanied by some irreversible energy loss, which cannot be avoided even at vanishing bias. Simple arguments based on the relation between entropy and information (von Neumann, 1966; Brillouin, 1956) lead to the "entropy" bound of this energy dissipation

$$W \geq W_e = k_B T \ln 2 \tag{8}$$

per each energy barrier removal without the proper preliminary bias.

So, the basic feature of PQ is its physical reversibility, which permits one to switch the device from one stable state to another without any uncontrollable rapid processes and so avoids the limit (8), if the parameters (λ and f) are changed in a proper way. This reversibility results in the low energy loss, which is now due to "viscous friction" alone (see Section 4).

Physical reversibility of an element does not necessarily mean the complete reversibility of the logic devices using these elements. Landauer (1961) and Keyes and Landauer (1970) have shown in detail that it is a lack of knowledge of the state of each element during the computation, which results in the energy loss (8) per each elementary logic operation. Let us have a look at the physical mechanism of this loss to find a way to avoid this dissipation.

3. REVERSIBLE COMPUTATION WITH PARAMETRIC QUANTRONS

First we give a brief description how one can transfer and process information using a set of PQs (Likharev, 1977). Figure 3 shows shift register (a) and elementary logical cells (b, c) consisting of PQs with the parameters λ changed by three-phase clock signals (d). Arrows show the inductive coupling of PQs which makes the "external force" f acting on a PQ, determined by the variable x in several adjacent PQs. For example, in the shift register (Figure 3a) each PQ is coupled weakly to two nearest

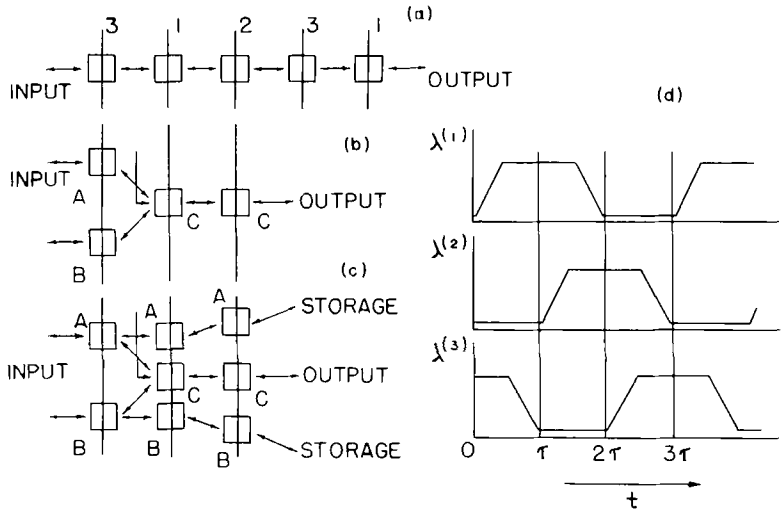

Fig. 3. Shift register (a), irreversible logic cell (b), and reversible logic cell (c) using PQs. Rectangles on a vertical "string" denote PQs controlled by the same clock current $I_c^{(i)}(t)$, so that their parameters $\lambda^{(i)}$ are changed synchronously. Functions $\lambda^{(1)}$, $\lambda^{(2)}$, and $\lambda^{(3)}$ are shifted in time to form a three-phase system (d), so that information is passed from left to right. Double-pointed arrows show inductive coupling between the PQ cells, while angled arrows show the permanent external biasing f_0.

neighbors, so that

$$f^{(i)} = k\left[x^{(i-1)} + x^{(i+1)}\right], \qquad k \ll 1 \tag{9}$$

Let a bit of information be stored in the left PQ cell at $t = 0$. It means that $\lambda^{(3)}$ is more than unity at the moment, so the energy barrier exists in this cell, and information is encoded by the cell state in one of the minima $x = \pm x_0$ of the two potential wells. This cell provides its neighbors, in particular, its right neighbor 1, with a weak biasing force $f_0 = kx_0$, resulting in small shift of the single energy minimum of the latter cell, because its energy barrier is suppressed ($\lambda^{(2)} < 1$). Now we start to increase $\lambda^{(2)}$, restoring the barrier in the region $x \approx 0$. In accordance with the discussion of Section 2, cell 1 will be led to a state close to $\pm x_0$, with the sign corresponding to that of x in the initial cell. In other words, the initial information is rewritten into the cell 1 now.

At this stage, one can "turn off" the initial cell 3 by decreasing $\lambda(3)$ and thus eliminating the energy barrier. Note that this cell is now biased by its right neighbor by a weak force $f \approx f_0 = kx_0$ of proper polarity, so the process proceeds just as shown in Figures 2a–2c. After the energy barrier in the left

cell is suppressed ($t = \tau$, Figure 3d), we have come just to the initial situation, but with the information shifted by one space period of the structure. Repeating the process in this three-phase manner, we can provide further shifts of information. Note that the process is completely reversible, i.e., there are no irreversible jumps of the variable x in any cell.

To perform logical operations, one can use the system of PQ cells shown in Figure 3b. Here, a logical cell $C^{(1)}$ is coupled not only with the output cell $C^{(2)}$, but also to two input cells $A^{(3)}$ and $B^{(3)}$. Moreover, an external "force" is applied to the cell:

$$f_C^{(1)} = k\left[x_C^{(2)} + s_A x_A^{(3)} + s_B x_B^{(3)}\right] + s_0 f_0,$$
$$|s_A| \approx |s_B| \approx |s_0| \approx 1, \qquad f_0 \approx k x_0, \tag{10}$$

the signs of s_A, s_B, s_0 being determined by the coupling polarities. f_0 represents a fixed signal, independent of the information content being processed.

The initial states of $x_A^{(3)}$ and $x_B^{(3)}$ can be of any polarity, depending on the information A and B which have come from the input in the manner described above. At the moment $t = 0$, the barrier in cell $C^{(1)}$ is turned off ($\lambda^{(1)} < 1$), and its only energy minimum is shifted to the left or to the right from the origin, according to the sign of the quantity

$$C = s_A A + s_B B + s_0, \qquad |A| = |B| = 1 \tag{11}$$

Turning this cell on (restoring its energy barrier) leads the cell to the state, corresponding to C (11). In this manner, any logical operation can be performed, depending on the signs of the coupling constants s_A, s_B, s_0. For example, the cell with $s_A = s_B = s_0 = 1$ provides the logical OR, while $s_A = s_B = -s_0 = 1$ provides the logical AND.

Note, that these logical operations are not reversible. In fact, when cells $A^{(3)}$ and $B^{(3)}$ are being turned off, one of them can be under the "wrong" bias provided by the logical cell $C^{(1)}$; so that the process follows the right column of Figure 2. This fact is directly connected to the logical irreversibility of the circuit (Keyes and Landauer, 1970). In fact, after the input cells are turned off, we cannot restore their initial state. The minimum energy dissipation W per operation is achieved as $k \to 0$ and equal just to W_c (8).

Bennett (1973) noticed that one can perform logically reversible computation by storing all the intermediate calculation results, and thus avoid the bound (8). In his arguments, he used a type of Turing machine, very far in its structure from the "usual" computers. We will demonstrate the possibility of reversible computation with the help of the more realistic circuits consisting of PQs.

Let us modify our logical system with two more cells $A^{(1)}$ and $B^{(1)}$ (Figure 3c), each coupled with one of the input cells, with the coupling factor $k' > k$. In this circuit, the information will not only be logically processed in the logical cell $C^{(1)}$, but also passed to these additional cells, when $\lambda^{(1)}$ is increased. Now, when $\lambda^{(3)}$ is being decreased and thus input cells $A^{(3)}$ and $B^{(3)}$ are being turned off, their back bias from the additional cells (always of the proper polarity) will overpower the back bias from the logical cell $C^{(1)}$ (of maybe wrong polarity). Thus, we avoid irreversible jumps in the input cells. Now the information A, B from the additional cells must be passed to a PQ-based memory for subsequent storage. Any attempt to erase the information would again lead to energy loss W_e (8), so after the calculation is over all the operations can be inverted to bring the computer and its memory to the original initial state with the exception of a memory containing the final result (Bennett, 1973). In the PQ-based computer, it can be achieved merely by changing the clock pulse ($\lambda^{(i)}$) order from $1 \rightarrow 2 \rightarrow 3$ to $3 \rightarrow 2 \rightarrow 1$.

Of course, reversible computation demands a greatly enlarged memory volume and is by no means practical nowadays, when typical energy consumption per bit is still much larger than $k_B T$, even for Josephson-effect-based devices (see, for example, Keyes, 1981). As a matter of principle, however, nothing in our reversible computation concept is absolutely unrealistic; therefore we need not take the entropy bound (8) into account in the further discussion.

4. CLASSICAL LIMITATIONS ON ENERGY DISSIPATION

The above discussion shows that devices like the parametric quantron can be an "ideal" computer element, in the sense of its low energy dissipation W. This enables us to discuss the lower bounds for W using this simple and realistic device.

Reviewing all operations of the information transfer and reversible processing described in Sections 2 and 3, one can assure oneself that they, in fact, consist of sequences of elementary operations of switching a single PQ, shown in the left column of Figure 2. Of course, the polarity of the bias depends on the processed information and can change from period to period, but W is evidently independent of the polarity and we can use, in particular, the process shown in Figure 2 for the discussion.

The only energy dissipation which results from the reversible switching of PQ is that due to viscous flow of the system and can be characterized by some viscosity factor η. Let us limit ourselves to the case of relatively slow operations, with the period τ large in comparison with some characteristic

relaxation time of the system, in which case the energy dissipation is minimum. This condition can be written in different ways for the systems with low and high damping:

$$\tau \gg \begin{cases} \hat{\tau} = \omega_c/\omega^2 \gg \omega^{-1}, & \text{at } \omega_c \gg \omega \text{ (low damping)} \\ \omega_c^{-1}, & \text{at } \omega_c \ll \omega \text{ (high damping)} \end{cases} \tag{12}$$

Here ω_c is the system "characteristic" frequency:

$$\omega_c = k/\eta \tag{13}$$

where k is the effective elasticity modulus:

$$k = d^2U/dx^2 \tag{14}$$

the latter derivative taken at the equilibrium point x, where $dU/dx = 0$.

For lightly damped systems, the relaxation time depends not only on k and η but also on the frequency ω of small oscillations around the equilibrium point:

$$\hat{\tau} = \omega_c/\omega^2 = m/\eta, \qquad \omega^2 = k/\eta \tag{15}$$

where m is the effective mass of the system. For the Josephson junctions, the characteristic frequency ω_c is of the order of $2\Delta/\hbar \sim 10^{12}$ s^{-1} (Δ is an energy gap of the superconductor being used), while ω is the "plasma frequency" dependent on the effective "mass"—normalized junction capacitance (see, for example, Likharev and Ulrich, 1978):

$$m = (\Phi_0/2\pi)^2 C$$

Coming back to the dissipated energy W, in the case (12) we can write

$$W = \int_0^T P\,dt = \eta \int_0^T (\dot{x})^2\,dt \tag{16}$$

The energy loss depends on the function $\dot{x}(t)$, and hence on the functions $\lambda(t)$ and $f(t)$. In any case, however, a reasonable estimate is given by

$$\dot{x} \approx x_0/\tau \qquad \text{so that } W \approx \eta x_0^2/\tau \tag{17}$$

where we omit numerical factors of the order of unity; x_0 is the maximum value of x during the operation cycle.

In equation (17), η and τ should be considered as given parameters, so that decreasing x_0 is the only way to reduce the dissipation. From Figures 1 and 2 one can readily see, however, that decreasing x_0 leads to lowering the energy barrier U_0 separating two static states (Figure 4), and consequently to increasing the probability p of the system spontaneous switching to the "wrong" state.

If p is kept very small ($p \ll 1$), it can be found as

$$p = \int_0^T \tau_L^{-1}\, dt \tag{18}$$

where τ_L is the lifetime of the "correct" state. For the thermally activated transitions "over" the energy barrier (arrow 1 in Figure 4) classical statistics yields the well-known result:

$$\tau_L^{-1} = (\omega_A/2\pi)\exp(-U_0/k_B T) \qquad \text{at } \tau_L \gg \hat{\tau}, \omega_c^{-1} \tag{19}$$

If the curvature d^2U/dx^2 of the $U(x)$ dependence is equal at the bottom of the well ($x = \hat{x}$) and at the top of the barrier ($x = 0$ in Figure 4), then

$$\omega_A = \begin{cases} \omega & \text{at } \omega_c \gg \omega \text{ (low damping)} \\ \omega_c & \text{at } \omega_c \ll \omega \text{ (high damping)} \end{cases} \tag{20}$$

Equations (18) and (19) show that τ_L^{-1} and hence p in fact increase exponentially with the lowering of U_0, and thus one should maintain U_0 and $\hat{x}$ to be large enough during the whole operation cycle. The exact relation between $\hat{x}$ and U_0 depends on the specific shape of the $U(x)$ function, but

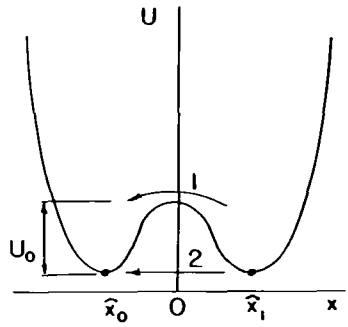

Fig. 4. Two processes leading to spontaneous switching of the state in a bistable element: (1) classical thermally activated "jump" over the energy barrier; (2) quantum-mechanical tunneling through the barrier.

for the realistic "smooth" functions [see equation (5) as an example] a reasonable estimate is given as

$$U_0 \approx kx_0^2/2 \tag{21}$$

Combining equations (17)–(21), one obtains that the energy dissipation W during a cycle must be larger than the "classical limit"

$$W_C \approx k_B T(\omega_c \tau)^{-1} \ln(\omega_A \tau p)^{-1} \tag{22}$$

The most essential feature of this limitation is the inverse proportionality of W_C to the cycle period, τ. This dependence reflects the process reversibility: $W \to 0$ as $\tau \to \infty$. Taking the values typical for the Josephson junctions, $T \approx 4$ K and $\omega_c \approx 10^{12}$ s^{-1} and fixing the reasonable values $p = 10^{-15}$, $\tau = 10^{-9}$ s, we get

$$W_C \sim 10^{-24} \text{ J} \tag{23}$$

a value well below the thermal fluctuation energy $k_B T \approx 10^{-22}$ J. Note that for "fast" nonlinear elements like Josephson junctions, quite reasonable computer speeds like $\tau \approx 10^{-9}$ s well satisfy the condition (12).

5. QUANTUM-MECHANICAL LIMITATIONS

The classical limit W_C (22) decreases when the temperature T is decreased. When T reaches the "quantum" value T_Q,

$$k_B T_Q \approx \begin{cases} \hbar\omega & \text{at } \omega_c \gg \omega \text{ (low damping)} \\ \hbar\omega_c & \text{at } \omega_c \ll \omega \text{ (high damping)} \end{cases} \tag{24}$$

the contribution of the macroscopic tunneling through the energy barrier (arrow 2 in Figure 4) to the probability p of spontaneous switching becomes of importance. The theory of this phenomenon for the systems with low damping is well understood presently (Likharev, 1981), so we will present here the results alone, assuming $T \ll T_Q$ for the sake of simplicity.

According to the above discussion, the operation speed is supposed to be within the following limits:

$$\omega^{-1} \ll \hat{\tau} \ll \tau \ll \tau_L \tag{25}$$

so the system is very close to its equilibrium state at the very bottom of the

energy well ($x \approx \hat{x}$), where its shape is nearly quadratic:

$$U(x) = k(x - \hat{x})^2/2 + \text{const} \tag{26}$$

From the quantum-mechanical point of view, the state of the system is very close to its basic (zero-point) state, and we can use well-known wave functions $\psi_0(x - \hat{x})$ of a harmonic oscillator for its description, with eigenenergy

$$E = E_0 = \hbar\omega/2 \ll U_0 \tag{27}$$

The exponential "tail" of this wave function penetrates inside the energy barrier and induces some small $\psi(x)$ in the other energy minimum. In the case of high energy barrier (27), analysis of this penetration can be conducted using the quasiclassical (WKB) wave function which coincides with ψ_0 in a transitional region $E \ll k(x - x)^2/2 \ll U_0$. Simple calculation shows a nonvanishing probability of tunneling only if two energy minima are almost equal:

$$|U(x_0) - U(x_1)| < E_0 \tag{28}$$

Within this region of small well asymmetry, the basic state lifetime is given by

$$\tau_L^{-1} = \left[\omega/2(\pi e)^{1/2}\right] \exp(-aU_0/\hbar\omega) \tag{29}$$

where the factor a changes from 6 to 12 depending on the specific shape of the barrier; for example, $a = 32/5$ for the symmetrical ($f = 0$) barrier (5) at not very high values of λ ($\lambda \gtrsim 1$).

Coming back to the energy dissipation itself, we note that according to equations (25) and (26) the system can be treated as an externally driven harmonic oscillator. The quantum statistics of such oscillators is well developed (see the monograph by Louisell, 1973, for example), and shows that one can again use equation (16) for W, having in mind that the low-temperature value of η can be, generally speaking, somewhat different from the high-temperature value.

Combining equations (17), (18), and (29), we arrive at the quantum-mechanical limitation on the energy dissipation of the PQ-type element with low damping:

$$W \geqslant W_Q, \qquad W_Q \approx (\hbar\omega/\omega_c\tau)\ln(\omega\tau p)^{-1}, \qquad \text{at } \omega_c \gg \omega \tag{30a}$$

The strict quantum statistics for nonlinear highly damped systems is not well developed yet, but preliminary estimates lead to the result

$$W_Q \approx (\hbar/\tau)\ln(\omega_c \tau p)^{-1} \qquad \text{at } \omega_c \ll \omega \tag{30b}$$

One sees that the quantum limitation for the highly damped systems is in qualitive agreement with the earlier estimates by Bledsoe (1961), Marko (1965), and Likharev (1977). The limit (30a) for the systems with low damping (say, PQs using tunnel Josephson junctions with their high capacitance) is, however, far lower than the estimates, because the factor $Q = \omega_c/\omega \gg 1$ can be much larger than the logarithmic factor.

This result does not contradict the so-called energy–time uncertainty condition (see, for example, Landau and Lifshitz, 1958)

$$\Delta E \times \Delta t \geq \hbar \tag{31}$$

First, as has been shown recently by Vorontsov (1980), one *can* measure the energy of a system with a precision ΔE better than is given by equation (31) during the time period Δt, if only one uses an optimum procedure. Second, what we have calculated in this paper is the energy *dissipation* W rather than the energy measurement *precision* ΔE. Let us discuss the relation between these two quantities.

The quantum-mechanical measurement (see, for example, Bohm, 1951) consists of at least two stages. At the first stage, the measured system is made to interact with some other quantum object (quantum instrument) and thus changes the state of this instrument. This first stage can be described by the Schrödinger equation and is reversible. At the second stage, we measure the state of quantum instrument by a classical instrument. According to present understanding (or present belief?), the second stage ("wave-packet reduction") cannot be described by the Schrödinger equation and is irreversible. The limitation on ΔE comes from the *second* stage of measurement.

Reviewing the above discussion of logic operations using PQ-type cells (say, simple information transfer to the adjacent cell), one notes the complete analogy of the operation with the *first* stage of quantum measurement, the adjacent cell acting like a quantum instrument. This process is completely reversible, and our main result (30) concerns only this process and thus has nothing to do with equation (31).

The interaction with a classical instrument is really needed at the stage of output information extraction from a "quantum" computer. This problem needs a special analysis, but we can note that any reasonable computer performs much more logical operations [when the result (30) is valid] than

those final read-out operations. So, the amount of energy dissipated during the latter operation is of relatively minor importance.

6. CONCLUSION

In contrast with some previous attempts to find the minimum energy dissipated in computation, we have used quite a realistic model of an elementary computer cell (a parametric quantron). Both the classical and quantum behavior of the device is well understood, which has enabled us to be quite confident in our results (22) and (30), giving, respectively, classical and quantum limitations on the energy dissipation during one logical operation.

Our approach to the problem, of course, leaves open whether it is possible to invent some novel device providing lower power consumption. If we limit ourselves to the quasistatic devices, where the computation can be stopped at any moment, without inducing an error, one can hardly get away from the above estimates. In fact, the only role of the parametric quantron in our discussion has been to demonstrate how a flexible bistable potential well could be physically realized. (Of course, some numerical factors can appear in the estimates if peculiar well shapes are taken into account.)

One can, however, argue that the above-mentioned condition of quasistatics is by no means compulsory, and that the information can be processed by some "dynamical" devices, where the cycle period can be shorter than the relaxation time. This problem is left for further analysis.

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